

Numbers of Rainy Days at Chennai, Kolkata, Mumbai, and New Delhi: Most Likely to Occur

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Abstract – Definition of probability based on the data on automatically happened outcomes, formulated in a recent study by the application of the logic behind concept of empirical probability, has been applied in estimating most likely number of rainy days at each of the four stations in India namely Chennai, Kolkata, Mumbai, and New Delhi to be occurred in each of the 12 months. It has been found from the study that rainfall is almost certain to be occurred in each of the months from June to November at Chennai, from May to October at Kolkata, from June to September at Mumbai and from June to August at New Delhi while rainfall is certain to be occurred in each of the months from September to November at Chennai, from June to September at Mumbai and from July to August at New Delhi. On the other hand, non-occurrence of rainy day or equivalently non-occurrence of rainfall is not certain in any month at any of the four stations.

Keywords: Automatically happened outcomes, probability, number of rainy days, most likely event, estimation.

1. INTRODUCTION

The theory of probability, which can be interpreted as the foundation of theory of statistics, has become essentially useful for scientific analysis of data not only in every branch of science but in every academic branch where research and study is done on the basis of data [9, 19, 23].

The evolution of theory of probability has progressed through five stages. Each of these stages can be regarded as an era of probability theory. These are

- (1) Prehistoric era,
- (2) Scientific thinking era,
- (3) Bernoullian Era,
- (4) Russian School era,
- and (5) Modern era [9, 16, 23, 37].
- On the other hand, the development has been progressing six approaches namely
- (1) Subjective Approach [4],
- (2) Intuitive Approach [34, 35, 46],
- (3) Classical Approach [6, 10, 12, 14, 15],



(4) Empirical Approach also termed as relative frequency approach or statistical approach [10, 13, 54, 55, 56],

(5) Axiomatic Approach [7,8,31,33]

and (6) Theoretical Approach [9, 13, 14, 15, 16, 17, 18, 23].

Subjective approach to probability is a biased one and hence unscientific while the concept of intuitive approach is not based on scientific logic. On the other hand, the other approaches are based on scientific logic. Probability is determined in an empirical approach by performing the associated experimentation while in classical approach probability is determined without performing the experimentation. Axiomatic approach is based on some conditions called axioms that are satisfied by probability, and it is silent about how to determine the value of probability. In theoretical approach, probability is defined in theoretically ideal situation and is determined in practically ideal situation by performing the associated experimentation.

In many real situations, experimentation need not be and/or cannot be performed but is automatically performed resulting in available outcomes. In order to handle this type of situation, a definition of probability has recently been formulated on the basis of automatically happened outcomes [28]. The definition has been formulated by the application of the logic behind concept of empirical probability. This definition has here been applied in estimating most likely number of rainy days at each of the four stations in India namely Chennai, Kolkata, Mumbai and New Delhi to be occurred in each of the 12 months. This paper is based on the findings obtained in this study. This study has been done due to the reason that there has not yet been any study on estimating most likely number of rainy days at a place though lot of studies had been done on various characteristics and behaviors of rainfall [1 - 3, 5, 11, 20 - 22, 24 - 32, 36 - 44, 47 - 53].

2. DEFINITION OF PROBABILITY FOR AUTOMATICALLY HAPPENED OUTCOMES

Let us use the standard notation P(E) to denote the probability of occurrence or happening of event E.

Probability has recently been defined on the basis of automatically happened outcomes of a natural phenomenon as follows: [28]:

Definition (1):

If in a set of N outcomes of a natural phenomenon already happened, an event E has occurred n times then the probability of occurrence of E is

the limiting value of the ratio
$$\frac{n}{N}$$
 as $N
ightarrow \infty$

i.e. P(E) can be approximated by the ratio N provided N is large

and thus the ratio N is a reasonable estimate of P(E) provided the size N of outcomes is reasonably large.

Definition (2):

If in a set of N outcomes of a natural phenomenon already happened, an event E with probability of occurrence P(E) has occurred n times then the number ⁿ of occurrence of the event E namely n is

the limiting value of the ratio N.P(E) as $N \rightarrow \infty$

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i.e. n can be approximated by N.P(E) provided N is large

and thus N.P(E) is a reasonable estimate of n provided the size N of outcomes is reasonably large.

Note:

(1) Content of Definition (1) is just the converse of that of Definition (2).

(2) Definition (1) provides a way of approximating/estimating the probability of occurrence of an event while Definition (2) provides a way of approximating/estimating the number of occurrence of the event.

(3) The happening of an event E is certain if P(E) = 1 and vice versa while the non-happening of E is certain if P(E) = 0 and vice versa.

(4) Happening or non-happening of an event E is uncertain if 0 < P(E) < 1 and vice versa. The uncertainty of happening or non-happening increases as P(E) increases from 0 or decreases from 1. The uncertainty is maximum when P(E) = 0.5 and vice versa.

(5) In a set of more than two mutually exclusive events, the event which has maximum probability (of occurrence) but less than 0.5 cannot be most likely to occur. In this case, the occurrence of either of the other events is more likely than this. Therefore, for an event to be most likely the probability (of occurrence) of it must be more than 0.5 and vice versa.

2.1. Most Likely Number of Rainy Days

Let us consider the number of rainy days in a period containing D number of days at a place. Let R denote the number of rainy days in the period occurred at the place. Then the possible values of R are

(If month is considered as the period, the possible value of D are 28, 29, 30, 31 depending upon length of the month.)

Suppose that R outcomes have already happened.

If out of R outcomes, d number of rainy days occurred r times

then by Definition (1), the probability of occurrence of r rainy days can be defined by

the limiting value of the ratio
$$\frac{R}{R}$$
 as $R \rightarrow \infty$

r

and can be approximated by this ratio provided R is large and thus can be reasonably estimated by this ratio provided the size R of outcomes is reasonably large.

The value of R that corresponds to the maximum value of probability of occurrence is the estimated most likely number of rainy days.

3. APPLICATION OF THE DEFINITION TO NUMERICAL DATA

Probability defined on the basis of the data on automatically happened outcomes, as stated above, has been applied in estimating most likely number of rainy days at each of the four stations in India namely Chennai, Kolkata, Mumbai and New Delhi to be occurred in each of the 12 months. The estimates have been computed



on the basis of data number of rainy days at these stations from 1969 onwards collected from Meteorological Department of India [28].

Estimates of the associated probabilities of number of rainy days at each of the five stations for each of the 12 months have been computed by the formulation as stated above. Then estimates of most likely numbers of rainy days have been computed from the corresponding values of probabilities obtained in the earlier stage.

It is to be noted that a single number (i.e. a point value) of most likely number of rainy day carries least the mostnificance in terms of acceptance in reality.

Moreover, the most likely numbers of rainy days with probability (of occurrence) less than 0.5 carry more uncertainty of occurrence. For this reason, estimates of most likely numbers of rainy days have been computed as interval values (closed intervals) and with probability (of occurrence) more than 0.9. The estimated values obtained have been shown in Table – 5.1. In addition to this, almost certain numbers of rainy days i.e. number of rainy days with probability (of occurrence) more than 0.99 have also been estimated which have been shown in Table – 5.2.

4. CONCLUSION

Most likely numbers of rainy days with probability (of occurrence) more than 0.9, shown in Table – 5.1, can be interpreted as or regarded as 90% confidence numbers of rainy days to be occurred. Similarly, almost certain numbers of rainy days with probability (of occurrence) more than 0.99, shown in Table – 5.21, can be interpreted as or regarded as 99% confidence numbers of rainy days to be occurred.

From the findings obtained in the study, the following facts have been observed in respect of the picture of rainfall at four stations namely Chennai, Kolkata, Mumbai and New Delhi:

(1) Rainfall is almost certain to be occurred in each of the months from June to November at Chennai, from May to October at Kolkata, from June to September at Mumbai and from June to August at New Delhi.

(2) Rainfall is certain to be occurred in each of the months from September to November at Chennai, from June to September at Kolkata, from July to September at Mumbai and from July to August at New Delhi.

(3) There exists neither certain nor almost certain non-rainy month at each of the four stations namely Chennai, Kolkata, Mumbai and New Delhi which means, the non-occurrence of rainy day or equivalently non-occurrence of rainfall is not certain in any month at any of the four stations.

(4) Non-rainy day is more likely to be occurred

- (i) in each of the months February, March & April at Chennai,
- (ii) in the month December at Kolkata,
- (iii) in each of the months January, February, March, April, May & December at Mumbai &
- (iv) in the month November at New Delhi.

It is to be mentioned that the findings obtained in this study are based on the assumption that data used in the analysis satisfy the condition(s) under which the definition of probability is valid. Thus the accuracy of findings is subject to the validity of this assumption.

At this stage, it can be concluded that the definition of probability formulated for automatically happened outcomes can be a convenient tool of determining most likely picture of rainfall at a place. Similar method



can be used in determining most likely picture of rainfall at other places not considered in this study. Thus, one problem for researchers, at this stage, is to go for study on finding the most likely picture of rainfall at the other places of the globe by the application of the definition of probability based on automatically happened outcomes. This type of study will carry significance in the interest of the globe.

One more point to be noted is that in this study attempt has been made on estimating/approximating most likely number of rainy days by the application of definition of probability. There is possibility that the same can be estimated by the application of the concept/definition of mathematical expectation [45]. Studying on the same by mathematical expectation can provide a way of finding the accuracy of the estimates of most likely number of rainy days.

5. TABLES OF FINDINGS

	Most likely number of rainy days at				
Month	Chennai	Kolkata	Mumbai	New Delhi	
January	0 - 4	0 - 2	0	0 - 3	
February	0 - 2	0 - 4	0	0 - 3	
March	0 - 1	0 - 6	0	0 - 3	
April	0 - 2	1-6	0	0 - 3	
Мау	0 - 4	2 – 11	0 - 3	0 - 4	
June	2 – 9	5 – 20	6 – 18	2 - 8	
July	3 - 12	15 – 22	16 – 28	5 – 18	
August	3 - 15	15 – 22	16-27	4 - 18	
September	4 - 13	8 - 18	6 – 25	2 – 11	
October	5 – 15	2 - 12	0 – 10	0 - 3	
November	5 – 18	0 - 3	0 - 3	0 – 2	
December	2 - 14	0 - 3	0 – 1	0 – 2	

Table -5.1: (Most likely number of rainy days with probability (of occurrence) more than 0.9)

Table -5.2: (Almost certain number of rainy days to occur)

	Almost certain occurrence of number of rainy at				
Month	Chennai	Kolkata	Mumbai	New Delhi	
January	0 – 7	0 - 3	0 – 1	0 - 4	
February	0 - 8	0 – 5	0 – 1	0 – 5	
March	0 - 3	0 - 7	0 – 1	0 – 5	

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April	0 - 4	0 - 9	0 – 1	0 - 6
Мау	0 – 5	1 – 16	0 – 7	0 - 6
June	1-9	5 – 22	5 – 19	1-9
July	1 – 12	11 – 22	16 – 28	4 – 21
August	1 – 15	15 – 22	12-27	3 – 18
September	2 – 13	7 – 19	5 – 25	0 – 11
October	3 – 21	1 – 12	0 – 10	0 - 4
November	4 - 21	0 - 4	0 – 5	0 - 4
December	0 – 18	0 - 4	0 - 2	0 - 6

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