# Arithmetic, Geometric and Harmonic Expectations: Expected Rainy Days in India

## Dhritikesh Chakrabarty

*Independent Researcher (Ex Associate Professor of Statistics, Handique Girls' College) Guwahati, Assam, India.*

------------------------------------------------------------------------------------

**Abstract –** An attempt has here been made on identifying the months which contain certain rainy days at 30 stations selected in India. Moreover, attempt has also been made on estimating expected number of rainy days in each of the months having certain rainy days at each of the selected 30 stations in India. The estimated values of the respective expected numbers have been computed by the application of the formulations of arithmetic expectation, geometric expectation and harmonic expectation.

## 1. INTRODUCTION

Rainfall is observed not to be occurred in every day or at every time at a place. But at a place, there may exist period(s) within which rainfall is certain to be happened. At a place, it may be required to know period(s) with certain rainfall for various purposes. Moreover, it may be required to know the amount of rainfall as well as number of rainy days in a period. Accordingly, an attempt has here been made on identifying the months which contain certain rainy days at 30 stations selected in India. Moreover, attempt has also been made on estimating expected number of rainy days in each of the months having certain rainy days at each of the selected 30 stations in India.

Mathematical expectation, a concept describing expected value of a random variable, was originally defined as the theoretical weighted arithmetic of mean of the possible values of the random variable with respective probabilities as weights [1–3 , 11 , 13 , 14 , 16 , 19 , 20]. Since a random variable assumes two or more possible values with respective probabilities it had to be defined as the weighted arithmetic mean of the possible values with respective probabilities as weights. This definition of mathematical expectation can be interpreted as arithmetic expectation [8]. This definition has already been applied in estimating expected number of rainy days in the context of India [9]. However, arithmetic expectation may not be suitable for determining expected value in every situation. Recently, two more definitions of expectation have been developed in order to measure expected value of a random variable which have been termed as geometric expectation and harmonic expectation [8] since these are based on geometric mean and harmonic mean respectively [4 , 12 , 15 ]. In this study all these three definitions of expectation have been applied in determining expected number of rainy days, in the context of India based on 30 locations/stations, in each of the months in which rainfall is certain to be occurred.

It is to be mentioned that in a non-rainy month (i.e. in a month having 0 (zero) rainy day), there may be rainfall due to some accidental or unnatural cause which is very rare. Thus the occurrence of 1 (one) rainy day in a month may be due to some accidental or unnatural cause. In this study therefore, a month has been regarded as certain rainy if this contains two or more rainy days with certainty (i.e. with probability 1).

In determining the expected number of rainy days, values of probabilities of possible numbers of rainy days are to be used as weights of the respective possible numbers as per the formulations of expectation. The approach/definition which was developed for determining the value of probability of an event is the empirical approach/definition of probability [10 , 16 –18] where the value is determined by performing the associated actual experimentation. But rainfall is not an experimentation which can actually be performed. It is a natural (or automatic) process/phenomenon and hence the empirical definition of probability cannot be applied in determining the value of probability of number of rainy days. Of course, the extended definition of empirical probability [5 , 6 , 7] developed in recent study can be suitably applied in this case.

## 2. Arithmetic, Geometric & Harmonic Expectations

Arithmetic expectation, geometric expectation and harmonic expectation of a discrete random variable are defined as follows:

If

$$x_1 , x_2 , \ldots , x_N$$

are real values assumed by a discrete random variable X with respective probabilities

$$P(X = x_1) = p_1 , P(X = x_2) = p_2 , \ldots , P(X = x_N) = p_N$$

then the arithmetic expectation of X, denoted by EA(X), is defined by

$$EA(X) = x_1 \cdot P(X = x_1) + x_2 \cdot P(X = x_2) + \ldots + x_N \cdot P(X = x_N)$$

i.e. $$EA(X) = x_1 \cdot p_1 + x_2 \cdot p_2 + \ldots + x_N \cdot p_N \qquad (2.1)$$

If the possible values assumed by X are strictly positive and real then the geometric expectation of X, denoted by EG(X), is defined by

$$EG(X) = x_1^{P(X = x_1)} x_2^{P(X = x_2)} \ldots x_N^{P(X = x_N)}$$

i.e. $$EG(X) = x_1^{p_1} x_2^{p_2} \ldots x_N^{p_N} \qquad (2.2)$$

If the possible values assumed by X are non-zero real then the harmonic expectation of X, denoted by EH(X), is defined by

$$EH(X) = \frac{1}{\frac{1}{x_1}P(X=x_1) + \frac{1}{x_2}P(X=x_2) + \ldots + \frac{1}{x_N}P(X = x_N)}$$

i.e. $$EH(X) = \frac{1}{\frac{1}{x_1}p_1 + \frac{1}{x_2}p_2 + \ldots + \frac{1}{x_N}p_N} \qquad (2.3)$$

Now suppose, the random variable X denotes the number of rainy days at a place in a month in which rainfall is certain to be occurred (i.e. probability of occurrence of two or more rainy days is 1).

Then the possible values assumed by X are strictly positive integral valued.

Hence all the three expectations of X, as mentioned above, exist in this situation.

Thus the values of arithmetic expectation, geometric expectation and harmonic expectation of number of rainy days occurring in a month can be estimated by the equations (2.1) , (2.2) & (2.3) respectively.

It is to be mentioned that

$$EA(X) > EG(X) > EH(X) \hspace{2cm} (2.4)$$

Moreover, number of rainy days is an integral vale.

Therefore, the common integral value nearest to $EA(X)$, $EG(X)$ & $EH(X)$ can be regarded as the expected value of number of rainy days in the month.

## 3. Expected Rainy Days in India

The formulations of arithmetic expectation, geometric expectation and harmonic expectation have been applied in estimating expected numbers of rainy days in the months which cottain certain rainy days at the 30 stations

Agartala , Ahmadabad , Allahabad , Amritsar , Bangalore , Bhopal , Bhubaneswar , Bhunter , Chennai , Guwahati , Hisar , Hyderabad , Imphal , Jaipur , Kolkata , Lucknow , Mumbai , Nagpur , New Delhi , Palam , Panjim , Patna , Pondicherry , Port Blair , Pune , Shillong , Tezpur , Trivandrum , Udaipur , Varanasi in India.

Determinations of the estimated values of numbers of rainy days have been carried out with the following steps:

1) At the first step, probability distribution of number of rainy days have been computed for each of the 12 months at each of the 30 stations on the basis of the data [5 , 9] on number of rainy days (month-wise) at the 30 stations.

2) At the second step, the months in which rainfall is certain to be occurred (i.e. probability of occurrence of two or more rainy days is 1) have been identified for all the 30 stations. The months identified as having certain rainy days have been listed in Table – 5.1.

3) In the next step, arithmetic expectation, geometric expectation & harmonic expectation have been computed by the formulas (2.1), (2.2) & (2.2) respectively.

4) In this step, the common integral value nearest to arithmetic expectation, geometric expectation & harmonic expectation has been identified corresponding to each of the months and at each of the stations. These common integral values are the respective desired estimates of expected numbers of rainy days. These estimates have been listed in Table – 5.1 within brackets after the respective rainy months.

## 4. CONCLUSION

Originally the term "Expectation" was introduced to mean the average of the possible vales of a random variable. Since a random variable assumes many possible values with respective probabilities, expectation had to be defined as the weighted average of the possible values with the respective probabilities as weights. Due to the acceptability of arithmetic mean as ideal measure of average, expectation was defined as the weighted arithmetic mean. However, defining expectation by weighted geometric mean and weighted

harmonic mean is also valid and each of these two definitions also corresponds to situation(s) where it is suitable to be applied.

It is to be mentioned that all these three definitions do exist in the situation where the possible valued assumed by a variable are all strictly positive. Hence, these three have been applied in estimating the expected number of rainy days in certain rainy month because the possible values of number of rainy days in certain rainy month are strictly positive integers.

From the estimated values in Table – 5.1, the following notable findings are observed:

(1) July and August are the two common months among the 30 stations which contain certain rainy days.

(2) January and February are the two common months among the 30 stations which contain rare rainy days.

(3) Pondicherry is the only station where the month December is certain rainy.

(4) Imphal is the only station where the month March is certain rainy.

(5) Imphal and Trivandrum are the two stations having maximum number (8) of certain rainy months.

(6) Ahmadabad, Amritsar, Hisar, and Jaipur are the four stations having minimum number (2) of certain rainy months.

(7) Variation of certain rainy months is visible among the 30 stations.

(8) Variation of expected number of rainy days is also visible in the same certain rainy months among the 30 stations.

In view of the finding (5), there is necessity of study on amount of rainfall in the certain rainy months in order to obtain a picture of level/degree of rainfall in the rainy months at the stations as well as to compare the level/degree at different stations.

## 5. Tables of Findings – Values Obtained from Computation

### Table –5.1:

Month having certain rainy days (with estimated expected number of rainy days)

| Station | Month (estimated expected number of rainy days) |
|---|---|
| Agartala | April (8) , May (13) , June (15) , July (15) , August (15) , September (11) , October (6) |
| Ahmadabad | July (11) , August (10) |
| Allahabad | July (11) , August (11) , September (8) |
| Amritsar | July (9) , August (7) |
| Bangalore | May (6) , June (6) , July (7) , August (10) , September (9) , October (8) |
| Bhopal | June (7) , July (14) , August (14) , September (7) |
| Bhubaneswar | June (10) , July (15) , August (15) , September (11) , October (7) |

| | |
|---|---|
| Bhunter | May (6) , June (4) , July (8) , August (8) , September (4) |
| Chennai | June (4) , July (6) , August (8) , September (7) , October (10) , November (10) |
| Guwahati | April (9) , May (12) , June (14) , July (17) , August (12) , September (10) , October (4) |
| Hisar | July (7) , August (6) |
| Hyderabad | June (7) , July (9) , August (10) , September (7) |
| Imphal | March (6) , April (9) , May (10) , June (15) , July (15) , August (12) , September (9) , October (6) |
| Jaipur | July (10) , August (9) |
| Kolkata | May (6) , June (12) ,  July (17) , August (16) , September (13) |
| Lucknow | July (11) , August (11) , September (8) |
| Mumbai | June (13) , July (22) , August (21) , September (13) |
| Nagpur | June (8) , July (13) , August (13) , September (8) |
| New Delhi | June (4) , July (10) , August (9) |
| Palam | June (3) , July (9) , August (9) |
| Panjim | June (21) , July (26) , August (24) , September (12) , October (5) |
| Patna | June (6) , July (14) , August (12) , September (10) |
| Pondicherry | July (4) , August (6) , September (6) , October (9) , November (11) , December (6) |
| Port Blair | May (15) , June (18) , July (18) , August (18) , September (17) , October (14) , November (12) |
| Pune | June (9) , July (12) , August (9) , September (7) |
| Shillong | April (8) , May (15) , June (18) , July (18) , August (15) , September (16) , October (8) |
| Tezpur | April (10) , May (12) , June (15) , July (16) , August (13) , September (11) , October (5) |
| Trivandrum | April (6), May (9), June (16), July (13), August (10), September (8), October (11), November (9) |
| Udaipur | June (4) , July (8) , August (9) |
| Varanasi | July (13) , August (12) , September (9) |

## REFERENCES

[1] András Simonovits (1992):  "Indexed Mortgages and Expectations: Mathematical Analysis and Simulation", Budapest: Institute of Economics, Hungarian Academy of Sciences.

[2] Broze, Laurence (1990): "Reduced Forms of Rational Expectations Models", Chur [Switzerland]: Harwood Academic Publishers.

[3] Brunnermeier Markus Konrad (2004): "Optimal Expectations", Cambridge, MA: National Bureau of Economic Research.

[4]  Coggeshall F. (1886): "The Arithmetic, Geometric, and Harmonic Means", The Quarterly Journal of Economics, 1(1), 83–86. https://doi.org/10.2307/1883111 .  https://www.jstor.org/stable/1883111 .

[5] Dhritikesh Chakrabarty (2023): "Definition of Probability Based on Already Happened Outcomes: Application in Identifying Rainy and Non-Rainy Period", Partners Universal International Innovation Journal (PUIIJ), 01(04), 259 – 267. www.puiij.com . DOI:10.5281/zenodo.8282811.

[6] Dhritikesh Chakrabarty (2023): "Extended Definition of Statistical Probability: Estimate of Probability Distribution of Rainy Days in Southern Part of India", Partners Universal International Innovation Journal (PUIIJ), 01(06), 148 – 158. www.puiij.com . DOI:10.5281/zenodo.10392846.

[7] Dhritikesh Chakrabarty (2024): "Extension of Statistical Definition of Probability: Expected Number of Rainy Days in Indian Context", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 11(1), 21384 – 21393. www.ijarset.com .

[8] Dhritikesh Chakrabarty (2024): "Idea of Arithmetic, Geometric and Harmonic Expectations", Partners Universal International Innovation Journal (PUIIJ), 02(01), 119 – 124. www.puiij.com . DOI:10.5281/zenodo.10680751.

[9] Dhritikesh Chakrabarty (2024): "Expected Rainy Days: Tendency of Rainfall in Indian Context", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 11(2), 21490 – 21500. www.ijarset.com .

[10] Gioacchino Di Paola, Alessandro Bertani, Lavinia De Monte and Fabio Tuzzolino (2018 ): "A Brief Introduction to Probability", J Thorac Dis.,10(2),1129 – 1132. doi: 10.21037/jtd.2018.01.28.

[11] Grandmont Jean-Michel (1988): "Stability, Expectations and Predetermined Variables", Stanford, Calif: Institute for Mathematical Studies in the Social Sciences, Stanford University.

[12] Huffman Carl (2014): "A History of Pythagoreanism", Cambridge University Press. p. 168. ISBN 978-1139915984.

[13] Lintao Guo (2014): "The Application of Probability Statistics to Solving the Practical Problems", Scholars Journal of Engineering and Technology, 2(3C), 444 – 446. www.saspublisher.com .

[14] McCallum, Bennett T. (2002): "Consistent expectations, Rational Eexpectations, Multiple-Solution Indeterminacies, and Least-SquaresLlearnability", Cambridge, MA: National Bureau of Economic Research.

[15] Mercer P. R. (2003): "Refined Arithmetic, Geometric and Harmonic Mean Inequalities", Rocky Mountain J. Math, 33(4), 1459 – 1464.

[16] Mood A. M., Graybill F. A. and Boes D. C. (1974: "Introduction to the Theory of Statistics (3rd ed.)", Section 2.2, McGraw-Hill. ISBN 0070428646. (available online Archived 2012-05-15 at the Wayback Machine).

[17] von Mises R. (1939): "Probability, Statistics and Truth", Mcmillan.

[18] von Mises R. (1941): "On the foundation of Probability and Statistics", Annals Mathematical Statistics, 12, 191 – 205.

[19] Yadav S. K. , Singh S. & Gupta R. (2019): " Random Variable and Mathematical Expectation", In: Biomedical Statistics, Springer, Singapore. https://doi.org/10.1007/978–981–32–9294–9_26.

[20] Wojciech Herer (1992): "Mathematical Expectation and Strong Law of Large Numbers for Random Variable with Values in a Metric Space of Negative Curvature, Probability and Mathematical Statistics, 13(1), 59 – 70.