Partners Universal International Research Journal (PUIRJ)
Volume: 05 Issue: 02 | April – June 2026 | ISSN: 2583-5602 | www.puirj.com

Combined Set of Several Sets of Observations: Cubic Mean

Dhritikesh Chakrabarty

Independent Researcher, Guwahati, Assam, India.

Abstract – An algebraic expression of the cubic mean of a set of observations, which is the combined set of several sets of observations in terms of the cubic means of the sets and the respective numbers of observations in the sets, has been derived for the purpose of computing the cubic mean of the combined set when the observations of the sets are unknown but the individual cubic means of the sets along with the respective numbers of observations are known. Derivation of the formula, along with a numerical example, has been presented in this article.

Keywords: Observations, Several Sets, Combined Set, Cubic Mean, Formula.

1. INTRODUCTION

Cubic Mean [10 , 13] which is one of the measures of average in addition to the common and widely used measure arithmetic mean, geometric mean & harmonic mean [2 , 15 , 16 , 17] is also important and applicable in some situations. Though the concept of cubic mean had been introduced and then applied in solving some problems [10 , 13], study on its properties are yet in infant stage. Of course, such study are in progress [9].

It occurs in some situations that the cubic means of several sets of observations are available but the observations are unavailable while the necessity in such situation is to find out the cubic mean of all the observations of the sets combined together. This type of situation is likely in meta-analysis [1 , 3 , 4 , 11 , 12 , 14]. In order to obtain a solution of such problem, an algebraic expression of the cubic mean of a set of observations which is the combined set of several sets of observations in terms of the cubic means of the sets and the respective numbers of observations in the sets has been derived here for the purpose of computing the cubic mean of the combined set when the observations of the sets are unknown but the individual cubic means of the sets along with the respective numbers of observations are known. Derivation of the formula, along with numerical example, has been presented in this article.

2. CUBIC MEAN OF A SET OF NUMBERS

Let us consider a set of N real numbers

$$a_1 , a_2 , \ldots\ldots , a_N$$

Cubic mean [10 , 13] of them, denoted by C($a_1 , a_2 , \ldots\ldots , a_N$), is defined by

$$C(a_1 , a_2 , \ldots\ldots , a_N) = \{\frac{1}{N} \sum_{i=1}^{N} a_i^3\}^{1/3} \qquad (2.1)$$

The definition implies that

$$\text{Sum of cubes of } a_1 , a_2 , \ldots\ldots , a_N = \{C(a_1 , a_2 , \ldots\ldots , a_N)\}^3 \qquad (2.2)$$

Partners Universal International Research Journal (PUIRJ)
Volume: 05 Issue: 02 | April – June 2026 | ISSN: 2583-5602 | www.puirj.com

Thus, if c(a) is the cubic mean of $a_1, a_2, \ldots, a_N$

then

$$\text{Sum of the cubes of } a_1, a_2, \ldots, a_N = n\{c(a)\}^3 \qquad (2.3)$$

Note:

The first principle [5] from which cubic mean is derived states that the cubic mean of the numbers

$$a_1, a_2, \ldots, a_N$$

is the value c(a) which satisfies the equation

$$a_1^3 + a_2^3 + \ldots + a_N^3 = \{c(a)\}^3 + \{c(a)\}^3 + \ldots + \{c(a)\}^3$$

← N terms →

Equation (2.3). is also the consequence of the definition of cubic mean from first principle.

3. CUBIC MEAN OF COMBINED SET

Suppose, there are k sets namely

$$S_1, S_2, \ldots, S_k$$

containing

$$n_1, n_2, \ldots, n_k$$

observations respectively such that cubic means of the respective sets are

$$c_1, c_2, \ldots, c_k$$

respectively.

If the observations in the k sets are combined together to form a single set S, then this set will contain $\sum_{i=1}^{k} n_i$ observations.

By the logic in equation (2.3),

$$\text{Sum of cubes of the observations in the set } S_1 = n_1 c_1^3,$$

$$\text{Sum of cubes of the observations in the set } S_2 = n_2 c_2^3,$$

....

$$\text{Sum of cubes of the observations in the set } S_k = n_k c_k^3.$$

This implies,

$$\text{Sum of cubes of the observations in the set } S = \sum_{i=1}^{k} n_i c_i^3$$

Therefore, by the logic in equation (2.1), cubic mean c of the observations in the set S will be

$$c = \left(\frac{1}{n}\sum_{i=1}^{k} n_i c_i^3\right)^{1/3} \qquad (2.4)$$

where $n = \sum_{i=1}^{k} n_i$.

This is the algebraic expression for computing cubic mean of a combined set of observations from a number of sets of observations as aimed at.

This formula has been stated in the form of a theorem below:

Theorem (3.1):

If $c_1, c_2, \ldots\ldots, c_k$ are cubic means of respective sets having $n_1, n_2, \ldots\ldots, n_k$ observations respectively then the cubic mean of the set of all observations in the k sets combined together is given by

$$c = \left(\frac{1}{n}\sum_{i=1}^{k} n_i c_i^3\right)^{1/3}$$

where $n = \sum_{i=1}^{k} n_i$.

In particular, if c_x is the cubic mean of

$$x_1, x_2, \ldots\ldots, x_m$$

& c_y is the cubic mean of

$$y_1, y_2, \ldots\ldots, y_n$$

Then the cubic mean c_{xy} of

$$x_1, x_2, \ldots\ldots, x_m, y_1, y_2, \ldots\ldots, y_n$$

is given by

$$c_{xy} = \left\{\frac{1}{m+n}(m.c_x^3 + n.c_y^3)\right\}^{1/3} \qquad (2.5)$$

Note (3.1):

Formula (2.4), which has also been mentioned in Theorem (3.1), can also be expressed as

$$c = \left(\sum_{i=1}^{k} w_i c_i^3\right)^{1/3} \qquad (2.6)$$

were

$$w_i = \frac{n_i}{\sum_{i=1}^{k} n_i}, \quad (i = 1, 2, \ldots\ldots, k)$$

This formula can sometimes be more convenient in computational works.

4. NUMERICAL EXAMPLE

Let us consider the three sets

$$S_1 = \{1, 3, 5, 7, 9\}, \quad S_2 = \{2, 4, 6, 8, 10\}, \quad S_3 = \{1, 4, 9, 16, 25\}$$

of observations s that the combined set S of observations is given by

$$S = \{1, 3, 5, 7, 9, 2, 4, 6, 8, 10, 1, 4, 9, 16, 25\}$$

Computing from the observations it is found that

$$\text{Cubic mean of } S_1 = 6.257324745675973559015414975796 1,$$

$$\text{Cubic mean of } S_2 = 7.1137866089801256120123092444778,$$

$$\text{Cubic mean of } S_3 = 16.009109396030575941683119496371$$

$$\& \text{ Cubic mean of } S = 11.620863496622715537443485228965$$

Now, applying the formula given by equation (2.4), it is also found that

$$\text{Cubic mean of } S = 11.620863496622715537443485228965$$

5. CONCLUSION

The algebraic expression of cubic mean of combined set of several sets of observations in terms of the cubic means & numbers of observations of the individual sets, established here, is a solution of the problem finding the cubic mean of the combined set when the observations of the sets are unknown but the individual cubic means of the sets along with the respective numbers of observations are known.

It is mentionable that each of the measures of average is required to be computed in this type of situation. Of course, the formulas for computing arithmetic mean, geometric mean, harmonic mean and quadratic mean in this type of situation were already derived in some studies [6 , 7 , 8] while the formula for computing cubic mean has been derived here. However, the formulas for computing the other measures of average in this type of situation are yet to be found out. This is one more problem of further study as continuation to the current study.

REFERENCES

[1] Adams, D. C., Gurevitch, J. & Rosenberg M. S. (1997): "Resampling Tests for Meta-Analysis of Ecological Data", Ecology, 78(4), 1277 – 1283.
[2] Bullen P. S. (2003): "The Arithmetic, Geometric and Harmonic Means", Handbook of Means and Their Inequalities, ISBN 978-90-481-6383-0, 60 – 174. doi:10.1007/978-94-017-0399-4_2.
[3] Borenstein M., Hedges L. V., Higgins J. P. T. & Rothstein, H. R. (2009): "Introduction to Meta-Analysis", John Wiley & Sons. Google Scholar.
[4] Card, N. A. (2016): "Applied Meta-Analysis for Social Science Research (paperback edition). Guilford Press. Google Scholar.
[5] Dhritikesh Chakrabarty (2020): "Definition / Formulation of Average from First Principle", Journal of Environmental Science, Computer Science and Engineering & Technology, Section C, (E-ISSN : 2278 – 179 X), 9(2), 151 – 163. www.jecet.org. DOI: 10.24214/jecet.C.9.2.15163] .

Volume: 05 Issue: 02 | April – June 2026 | ISSN: 2583-5602 | www.puirj.com

[6] Dhritikesh Chakrabarty (2025): "Combined Set of Several Sets of Observations: Harmonic Mean", Partners Universal International Innovation Journal (PUIIJ), 3(1), 49 – 53. www.puiij.com . DOI:10.5281/zenodo.14949601.

[7] Dhritikesh Chakrabarty (2025): "Combined Set of Several Sets of Observations: Quadratic Mean", International Journal of Advanced Research in Science, Engineering and Technology, 0328), 12(5), 23444 – 23452. www.ijarset.com.

[8] Dhritikesh Chakrabarty (2025): "Combined Set of Several Sets of Observations: Geometric Mean", Partners Universal International Innovation Journal (PUIIJ), 3(3), 69 – 73. www.puiij.com. https://doi.org/10.5281/zenodo.15757409.

[9] Dhritikesh Chakrabarty (2025): "Cubic Mean of Cubic Means of Possible Subsets of a Set of Numbers", International Journal of Advanced Research in Science, Engineering and Technology, (ISSN: 2350 – 0328), 12(11), 23988 – 23996. www.ijarset.com.

[10] Eichelsbacher P. (2024): "Stein's Method and a Cubic Mean-Field Model", J Stat Phys, 191, 163. https://doi.org/10.1007/s10955-024-03373-x.

[11] Gurevitch, J., Koricheva, J., Nakagawa, S., & Stewart, G. (2018): "Meta-Aanalysis and the Science of Research Synthesis", Nature, 555(7695), 175 – 182.

[12] Hedges, L.V. & Olkin, I. (1985): "Statistical Methods for Meta-Analysis", Elsevier Science. http://qut.eblib.com.au/patron/FullRecord.aspx?p=1901162. Google Scholar.

[13] Stephan Baier and Matthew P. Young (2010): "Mean values with cubic characters", Journal of Number Theory", 130(4), 879 - 903. DOI: 10.1016/j.jnt.2009.11.007.

[14] Terri D. Pigott (2012): "Advances in Meta Analysis", Springer New York, NY. DOI: https://doi.org/10.1007/978-1-4614-2278-5.

[15] Weisstein Eric W.: "Arithmetic Mean", From MathWorld--A Wolfram Resource. https://mathworld.wolfram.com/ArithmeticMean.html.

[16] Weisstein Eric W.: "Geometric Mean", From MathWorld--A Wolfram Resource. https://mathworld.wolfram.com/GeometricMean.html.

[17] Weisstein Eric W.: "Harmonic Mean", From MathWorld--A Wolfram Resource. https://mathworld.wolfram.com/QuadraticMean.html.